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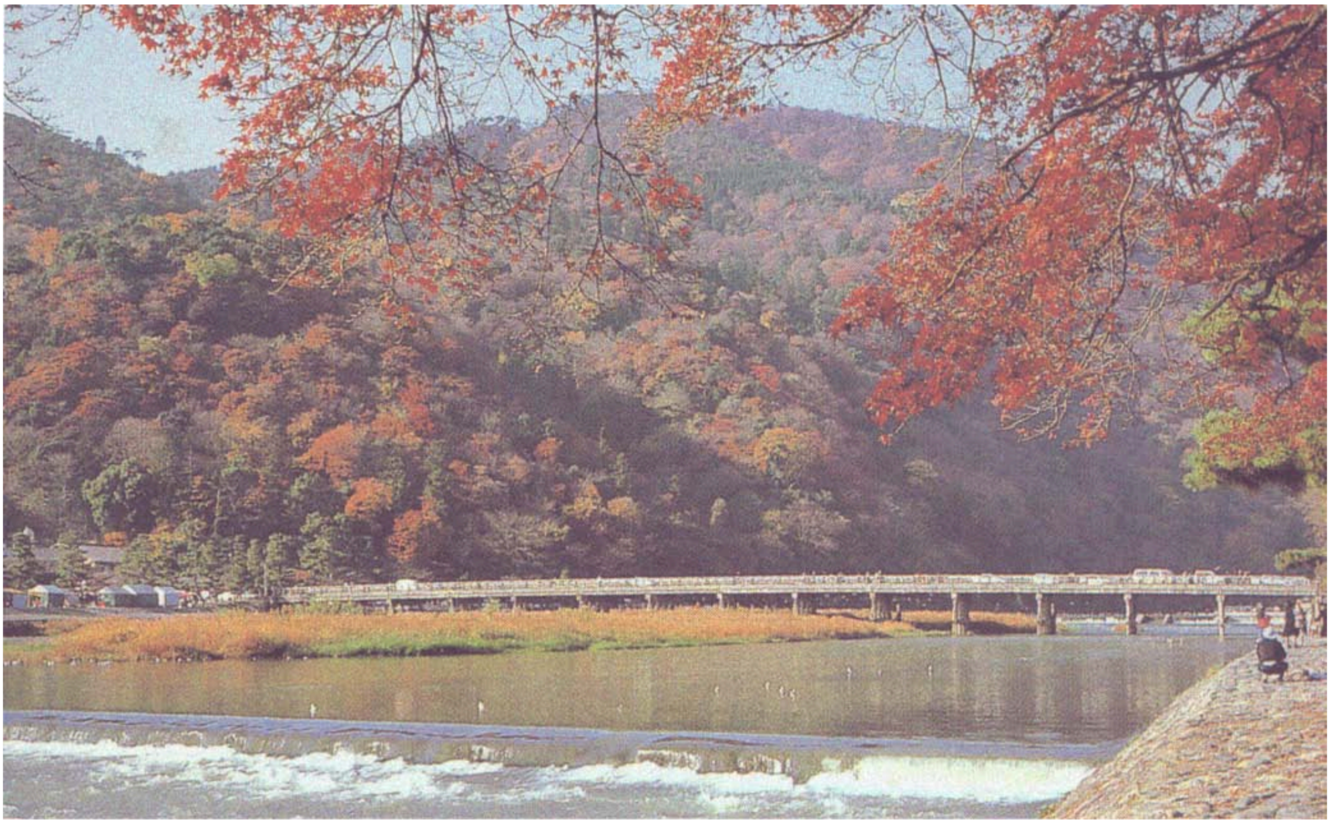

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SCALE DEPENDENT STATISTICAL PRECIPITATION ANALYSIS

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abstract

Areal precipitation is a difficult meteorological element to be analyzed. High quality data of precipitation are necessary for many scientific applications in hydrology, environmental and agricultural meteorology, often in real time or in form of forecasts. Because of the errors involved there is a need for a high quality analysis of precipitation. Therefore, first the statistical structure of precipitation fields on different space and time scales is discussed. Second, a statistical interpolation technique is applied to analyse meso-scale and large-scale precipitation fields over Europe. The data used are from rain gauge and weather radar networks. A comparison with fields from the *European Centre for Medium-Range Weather Forecasts* (ECMWF) shows a good agreement between precipitation fields from analysis and model forecast.

1. INTRODUCTION

For the spatial analysis of irregularly distributed precipitation data many methods have been proposed and applied to rainfall fields. These are the *nearest neighbour method*, the *arithmetic mean*, *spline surface fitting*, *interpolation based on empirical orthogonal functions* and statistical interpolation techniques. The latter are the so-called *optimum interpolation* or *optimal averaging* (Gandin, 1993) and the *kriging methods* (Krige, 1981). These methods have been developed since the early 60s in both meteorology and hydrology. Today statistical interpolation methods are state of the art; objective comparisons have pointed out their advantage with respect to other methods (Creutin and Obled, 1982).

The *optimum interpolation* and *kriging* techniques are based upon the knowledge of the statistical structure function (semivariogram) or the correlation function. Here the correlation function is used. The correlation function of precipitation depends primarily on the considered space-time aggregation and can be characterised by decorrelation distances (Zawadzki, 1973). With the knowledge of the correlation function a scale dependent precipitation analysis was performed for a model domain over Central Europe.

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2. METHOD

The statistical interpolation methods are linear or nonlinear, unbiased, least-squares spatial interpolation techniques. They are advanced applications of *Gauss' Theory of Errors* and therefore optimal in a statistical sense. Because of the assumptions of homogeneity and isotropy, which have to be made in the practical implementation, they do not necessarily yield optimal results. That is the reason why various assumptions have been proposed in the past, which lead to different, more or less expensive, interpolation methods. Here, for the objective analysis of the precipitation fields, the well known methods of ordinary block kriging (large scale analysis) and ordinary co-kriging (meso-scale analysis) are used.

2.1 Kriging of rain gauge data

Assuming n contemporary gauge observations of the precipitation process $Z_g(\mathbf{u})$, $\mathbf{u} \equiv (x, y)$, at points $i = 1, 2, ..., n$, the areal averaged precipitation $\widehat{Z}(\mathbf{u}_o)$ can be estimated from the linear combination

$$\widehat{Z}\left(\mathbf{u}_o\right) = \sum_{i=1}^{n} \lambda_{gi}\, Z_g\left(\mathbf{u}_i\right) \tag{1}$$

The weights λ_{gi} can be obtained under the condition of minimizing the estimation variance (mean square interpolation error) from the following system of linear equations

$$\left|\begin{array}{cccc} R\left(\mathbf{u}_1, \mathbf{u}_1\right) & ... & R\left(\mathbf{u}_1, \mathbf{u}_n\right) & 1 \\ ... & ... & ... & 1 \\ R\left(\mathbf{u}_n, \mathbf{u}_1\right) & ... & R\left(\mathbf{u}_n, \mathbf{u}_n\right) & 1 \\ 1 & ... & 1 & 0 \end{array}\right| \cdot \left|\begin{array}{c} \lambda_{g1} \\ ... \\ \lambda_{gn} \\ \mu_g \end{array}\right| = \left|\begin{array}{c} R\left(\mathbf{u}_1, \mathbf{u}_o\right) \\ ... \\ R\left(\mathbf{u}_n, \mathbf{u}_o\right) \\ 1 \end{array}\right| \tag{2}$$

where $R(\mathbf{u}_i, \mathbf{u}_j)$ are the correlations between rain gauge observations at the locations $\mathbf{u}_i$ and $\mathbf{u}_j$, $R(\mathbf{u}_j, \mathbf{u}_o)$ are the correlations between the rain gauge observations at the locations $\mathbf{u}_j$ and the grid area integrated true precipitation process $Z(\mathbf{u}_o)$ and μ_g is a scalar Lagrangian multiplier. Detailed descriptions of the method can be found in the literature (e.g. Wackernagel, 1995).

The system (2) can be written in short form by using the gauge-gauge correlation matrix $\mathbf{R_{gg}}$, the gauge-area correlation vector $\mathbf{R_{go}}$ and the vector of the weights to be estimated $\lambda_\mathbf{g}$:

$$\begin{vmatrix} \mathbf{R_{gg}} & 1 \\ 1 & 0 \end{vmatrix} \cdot \begin{vmatrix} \lambda_\mathbf{g} \\ \mu_g \end{vmatrix} = \begin{vmatrix} \mathbf{R_{go}} \\ 1 \end{vmatrix} \tag{3}$$

Thus, if the correlations are known, the vector $\lambda_\mathbf{g}$ can be calculated from Eq. (3) and the linear combination (1) for the area averaged precipitation can be evaluated.

The empirical estimation of the correlations $\mathbf{R_{gg}}$ and $\mathbf{R_{go}}$ for precipitation processes of different scales will be discussed in the next section.

2.2 Co-kriging of rain gauge and radar data

Because of the sparse density of rain gauge networks, the meso-scale analysis can be improved by using additional information from weather radar networks. In this case the linear combination for co-kriging of rain gauge data $Z_g(\mathbf{u})$ and radar data $Z_r(\mathbf{u})$ can be written as

$$\widehat{Z}\left(\mathbf{u}_o\right) = \sum_{j=1}^{n_r} \lambda_{rj}\ Z_r\left(\mathbf{u}_j\right) + \sum_{i=1}^{n_g} \lambda_{gi}\ Z_g\left(\mathbf{u}_i\right) \tag{4}$$

The weight vectors $\lambda_\mathbf{r}$ and $\lambda_\mathbf{g}$ that need to be estimated follow from the system of linear equations

$$\begin{vmatrix} \mathbf{R_{rr}} & \mathbf{R_{rg}} & 1 & 0 \\ \mathbf{R_{gr}} & \mathbf{R_{gg}} & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{vmatrix} \cdot \begin{vmatrix} \lambda_\mathbf{r} \\ \lambda_\mathbf{g} \\ \mu_r \\ \mu_g \end{vmatrix} = \begin{vmatrix} \mathbf{R_{ro}} \\ \mathbf{R_{go}} \\ 0 \\ 1 \end{vmatrix} \tag{5}$$

with the radar-radar correlation matrix $\mathbf{R_{rr}}$, the gauge-gauge correlation matrix $\mathbf{R_{gg}}$, and the gauge-radar crosscorrelation matrix $\mathbf{R_{gr}}$. Since the true area integrated precipitation is unknown, the gauge-area correlation vector $\mathbf{R_{go}}$ and the radar-area correlation vector $\mathbf{R_{ro}}$ are approximated from $\mathbf{R_{rr}}$ and $\mathbf{R_{gg}}$ (Krajewski, 1987).

Analogous to the kriging equation (3) the vectors $\lambda_\mathbf{r}$ and $\lambda_\mathbf{g}$ can be obtained from the co-kriging equation (5) if the correlations $\mathbf{R_{rr}}$, $\mathbf{R_{gg}}$ and $\mathbf{R_{gr}}$ are known.

3. STATISTICAL BACKGROUND

The statistical structure of precipitation is usually described by space and time correlation functions. Because precipitation is a complex physical phenomenon, it is not possible to fully describe precipitation processes by just one statistical model. Ideally, precipitation events should be classified by type e.g., stratiform or convective precipitation. Each type of precipitation can then be assumed to be a part of an ensemble of homogeneous realizations with defined statistical properties. The large- and meso-scale precipitation over Europe contains stratiform and convective components of different intensities and compositions at the same time. Moreover, it is hardly possible to separate them. For this reason the analysis of the precipitation fields was performed by the use of only a single correlation function for the whole model domain.

3.1 Empirical estimation of correlation functions

We assume that there exist n contemporary observations of the precipitation process $Z(\mathbf{u})$ at the locations $\mathbf{u}_i \equiv (x_i, y_i)$, $i = 1, 2, ..., n$. Further, if there exist multiple realizations $t = 1, 2, ..., T$ of this process $Z(\mathbf{u})$, then the correlation can be defined as

$$R(\mathbf{u}_i, \mathbf{u}_j) = \frac{\sum_{t=1}^{T} \left[Z_t(\mathbf{u}_i) - m(\mathbf{u}_i)\right]\left[Z_t(\mathbf{u}_j) - m(\mathbf{u}_j)\right]}{\sqrt{\sum_{t=1}^{T} \left[Z_t(\mathbf{u}_i) - m(\mathbf{u}_i)\right]^2 \sum_{t=1}^{T} \left[Z_t(\mathbf{u}_j) - m(\mathbf{u}_j)\right]^2}} \tag{6}$$

where $m(\mathbf{u}_i)$ is the mean of the precipitation process at the location $\mathbf{u}_i$.

Assuming that the precipitation process is homogeneous and isotropic within the model domain, then the correlations $R(\mathbf{u}_i, \mathbf{u}_j)$ depend only on the interstation distance ρ_{ij}; they are independent of the geographical locations of the stations. The result of the empirical analysis of the correlation coefficients is a scatter plot of correlations. To identify the structure of precipitation a hypothesis about the theoretical model had to be made. For the precipitation over Europe the nonlinear model $R(\rho) = c_1\, exp(-c_2\, \rho^{c_3})$ with 3 coefficients c_1, c_2 and c_3 was chosen. Note that the exponential correlation function has the required property to be positive definite.

3.2 Scale dependence of correlation functions

For a simple characterization of the correlation function a typical distance, the so-called decorrelation distance, will be used. It is defined as the distance for which the correlation decreases to 1/e. From the curve of the correlation function one can infer the type of precipitation. A strong decrease of the correlation function, corresponding to a short decorrelation distance, is characteristic for convective type precipitation. A weak decrease, corresponding to a wide decorrelation distance, is typical for stratiform precipitation.

The dependence of the correlation functions on the scale is summarized in Fig 1. If one considers the accumulation time of precipitation, which corresponds to a typical extension of the model domain, then typical decorrelation distances are 2 km for 1-minute (Witternigg et al., 1993), 10 km for 15-minute (Zawadzki, 1973), 50 km for hourly (Fig. 3), 200 km for 12-hourly and daily (Fig. 3 and Stol, 1972), 300 km for monthly (Berndtsson, 1987) and 500 km for annual accumulated values of precipitation (Berndtsson, 1987).

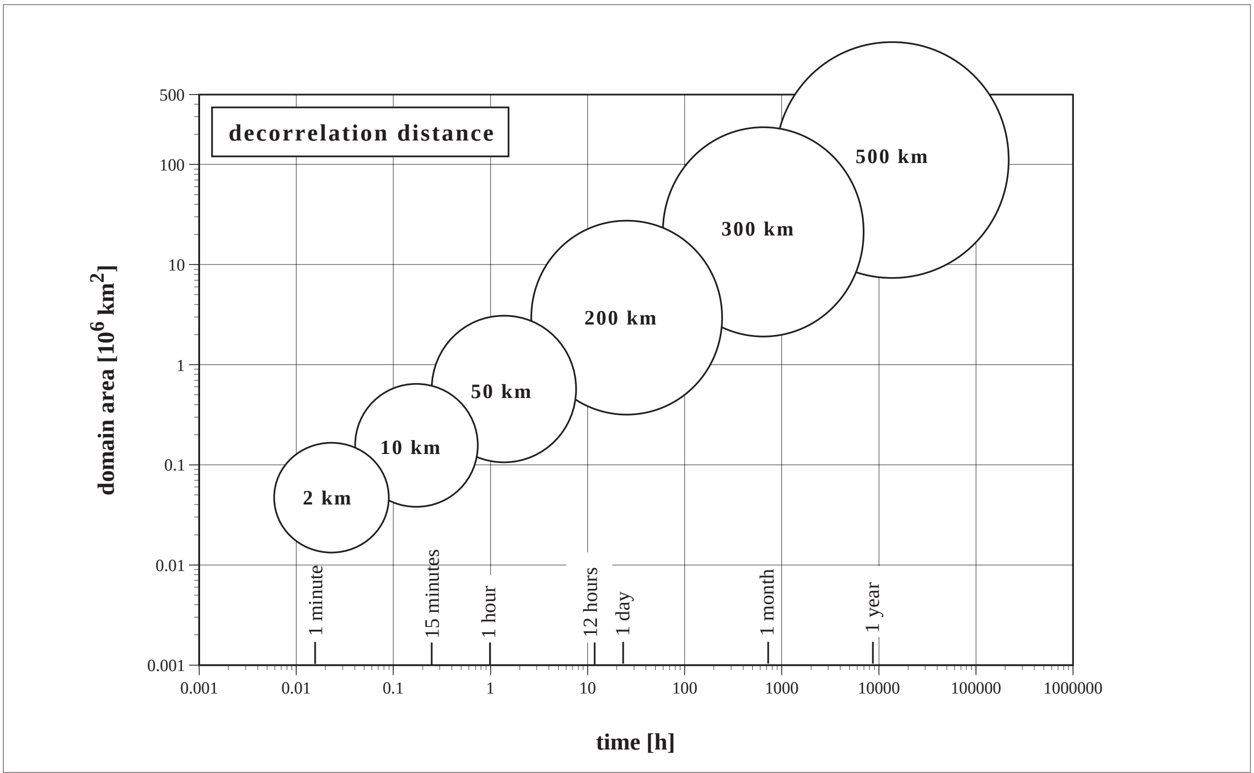

Figure 1. Scale dependence of correlation function used for statistical analysis of precipitation, represented here by decorrelation distance for increasing spatial and temporal resolution.

3.3 Typical correlation functions over Europe

The kriging and co-kriging techniques have been used to analyze large- and meso-scale precipitation over Central Europe. Fig. 2 shows the spatial correlation function for large-scale precipitation in September 1995.

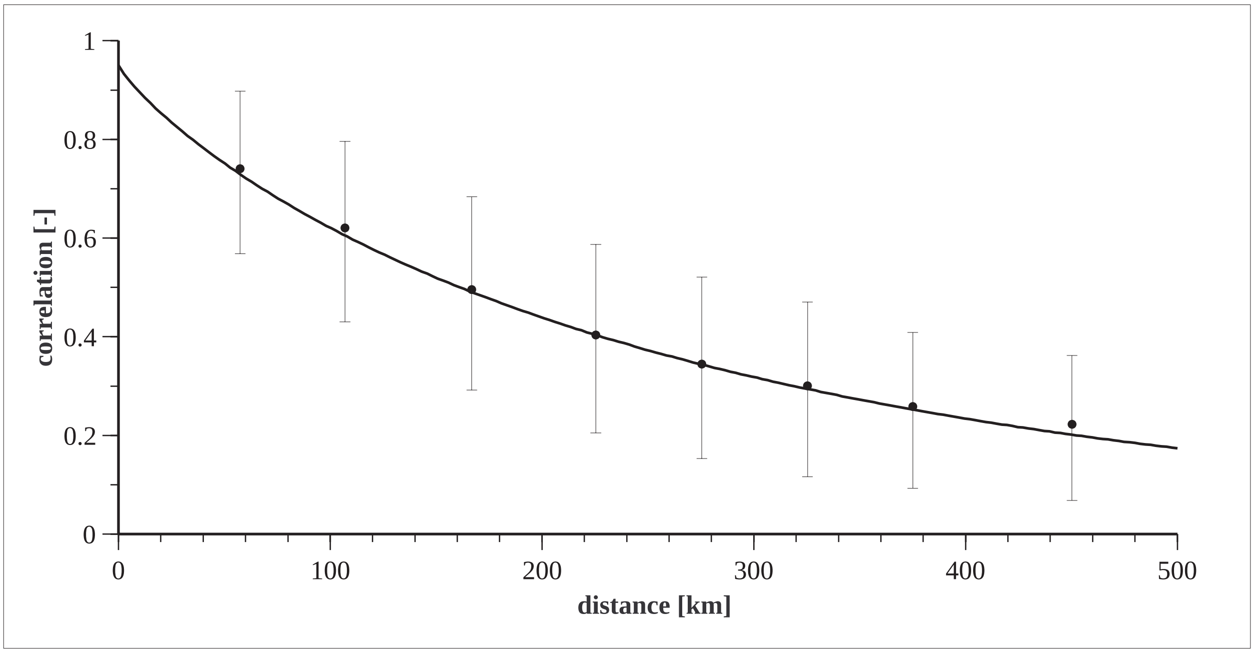

Figure 2. Correlation coefficients estimated from multiple realisations with standard deviations and fitted correlation function for September 1995. Temporal resolution 12 hour, model domain $7 \cdot 10^6$ km^2 (see Fig. 4).

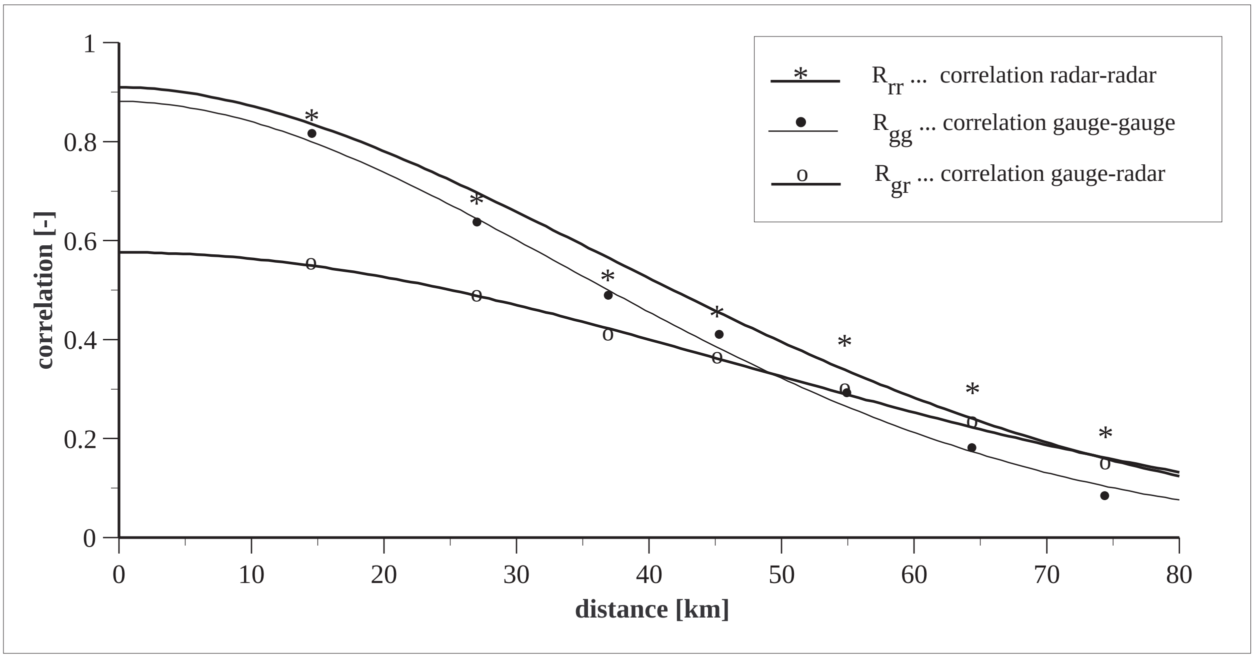

Figure 3. Correlation functions estimated from gridded rain gauge and radar data for 11 May 1992, 1500 UTC. Temporal resolution 1 hour, spatial resolution 12.5 km (model domain 30 000 km^2).

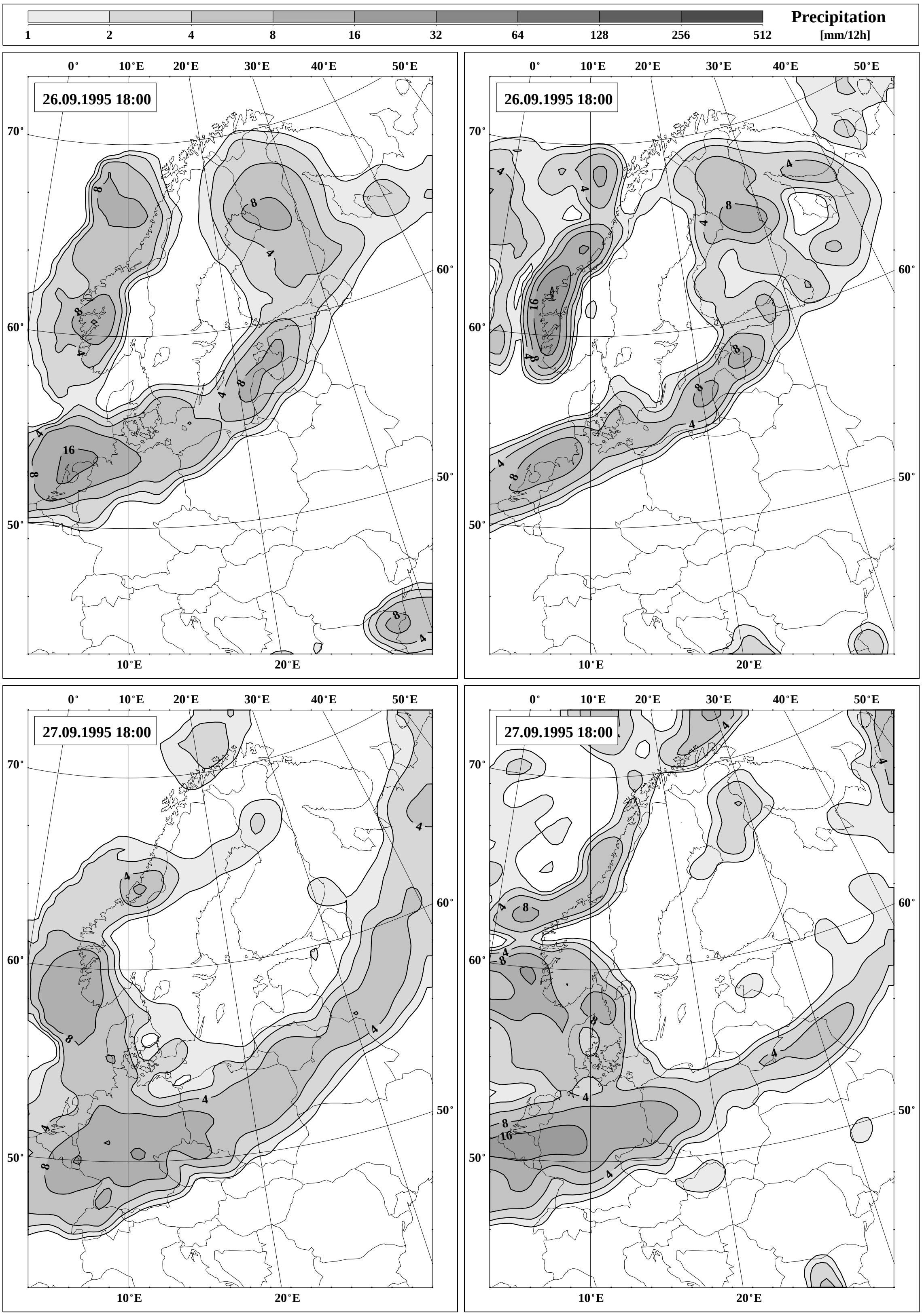

Figure 4. Objectively analyzed precipitation fields (left) vs. 18-30 hour forecast fields from ECMWF (right). Units are in mm/12h.

The fitted function $R(\rho) = 0.95\ exp(-0.008\,\rho^{0.86})$, was estimated from 60 realizations of 12-hourly accumulated precipitation observations over Central Europe. Fig. 3 shows the meso-scale autocorrelation functions of gauge and radar data together with the cross-correlation function of gauge and radar data for a single realization (11 May 1992, 1500 UTC) over Austria. Input data are 1-hourly accumulated, gridded precipitation data from about 100 climatological stations and 4 weather radars. The fitted functions are $R(\rho) = 0.91\ exp(-0.0006\,\rho^{1.85})$ for the radar-radar correlation, $R(\rho) = 0.88\ exp(-0.0006\,\rho^{1.90})$ for the gauge-gauge correlation and $R(\rho) = 0.58\ exp(-0.0002\,\rho^{2.00})$ for the gauge-radar correlation.

4. RESULTS

The results of the analysis are gridded large-scale precipitation fields with a grid distance of 55 km and a temporal resolution of 12 hours (Fig. 4, left) and meso-scale precipitation fields with a spatial resolution of 12.5 km and a temporal resolution of 1 hour (not shown). First verifications with model forecasts (Fig. 4, right) show a good agreement between analysis and forecast in both the pattern and the quantity of precipitation.

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